

Quantum Münchhausen effect in tunneling

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Abstract

It is demonstrated that radiative corrections increase tunneling probability of a charged particle.

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Famous baron von Münchhausen saved himself from a swamp pulling his hairs by the hands of his own [1]. According to classical physics, such a feat seems to be impossible. However, we live in a quantum world. In a tunneling of a charged particle, the head of the particle wave function can send a photon to the tail which absorbs this photon and penetrates the barrier with enhanced probability. Obviously, such a photon feedback should work in the two-body tunneling where the first particle, while continuing to be accelerated by the potential after the tunneling, can emit a (virtual) photon that increase energy of the second particle and its tunneling probability. The Münchhausen mechanism may be helpful in the tunneling of a composite system. It is related to phonon assisted tunneling but does not require any special device being always provided by the interaction of a charged particle with the radiation field.

The interaction of a tunneling object with other degrees of freedom of the system and the influence of this interaction on the tunneling probability for a long time was a topic of intensive studies initiated by Caldeira and Leggett [2]. Their general conclusion, in agreement with intuitive arguments, was that any friction-type interaction suppresses the tunneling. At the same time, it was realized that such an interaction leads to distortions of the barrier which can be helpful in endorsing the tunneling. The simplest effect is associated with the zero-point vibrations of the source responsible for the existence of the barrier. This is important for the probabilities of subbarrier nuclear reactions as pointed out by Esbensen [3]. In the last decade, many experimental and theoretical efforts were devoted to the understanding of related aspects of subbarrier reactions, see the recent review [4] and references therein. Below we discuss the interaction of a charged tunneling object with the electromagnetic field which always accompanies motion of the object.

Formally speaking, we are looking for the effects of radiative corrections on the single-particle tunneling. These effects can be described by the Schrödinger equation with the self-energy operator:

$$\hat{H}\Psi(\mathbf{r}) + \int \Sigma(\mathbf{r}, \mathbf{r}'; E)\Psi(\mathbf{r}')d^3r' = E\Psi(\mathbf{r}) \quad (1)$$

where \hat{H} is the unperturbed particle hamiltonian, which includes a barrier potential, and $\Sigma = M - i\Gamma/2$ is the complex nonlocal and energy-dependent operator determined by the coupling to virtual photons and a possibility of a real photon emission. The “photon hand” here connects two points \mathbf{r} and \mathbf{r}' of the same wave function. In the one-photon approximation the self-energy due to the interaction with the transverse radiation field can be written as

$$\Sigma(\mathbf{r}, \mathbf{r}'; E) = \sum_{\mathbf{k}, \lambda} |g_{\mathbf{k}}|^2 \sum_n \frac{\langle \mathbf{r} | (\hat{\mathbf{p}} \cdot \mathbf{e}_{\mathbf{k}\lambda}) e^{i\mathbf{k}\hat{\mathbf{r}}} | n \rangle \langle n | (\hat{\mathbf{p}} \cdot \mathbf{e}_{\mathbf{k}\lambda}^*) e^{-i\mathbf{k}\hat{\mathbf{r}}} | \mathbf{r}' \rangle}{E - E_n - \omega_{\mathbf{k}} - i0}. \quad (2)$$

Here we sum over unperturbed stationary states $|n\rangle$; $\hat{\mathbf{r}}$ and $\hat{\mathbf{p}}$ are the position and momentum operators, respectively; the photons are characterized by the momentum \mathbf{k} , frequency $\omega_{\mathbf{k}}$ and polarization λ ; the polarization vectors $\mathbf{e}_{\mathbf{k}\lambda}$ are perpendicular to \mathbf{k} so that the momentum operators commute with the exponents. The normalization factors are included into $g_{\mathbf{k}} \propto \omega_{\mathbf{k}}^{-1/2}$. The relativistic generalization of (2) is straightforward.

The hermitian part M of the self-energy operator is given by the principal value integral over photon frequencies in (2). The expectation value of M is responsible for the Lamb shift of bound energy levels. It contains also the mass renormalization for a free particle which should be subtracted. Our problem is different from the energy shift calculation for bound states since we are interested in the change of the wave function of the tunneling particle. However we can use some features of the conventional approach. As well known from the Lamb shift calculations, one can use different approximations in the two regions of integration over the photon frequency ω . In the nonrelativistic low-frequency region, $\omega < \beta m$, where the parameter $\beta < 1$ is chosen in such a way that typical excitation energies of a particle in the well δE are smaller than βm (in the hydrogen Lamb shift problem a fine structure constant α can play the role of the borderline scale parameter), it is possible to neglect the exponential factors in (2). The high-frequency contribution to M , where the potential can be considered as a perturbation to free motion, has been calculated, e. g. in Ref. [5]. The two contributions match smoothly at $\omega = \beta m$.

It is easy to estimate the mass operator M with logarithmic accuracy. After summation

over polarizations and standard regularization [5], the low frequency part of the operator M can be written as

$$\hat{M}(E) = \frac{2Z^2\alpha}{3\pi m^2} \int d\omega \sum_n \hat{\mathbf{p}}|n\rangle \frac{E - E_n}{E - E_n - \omega} \langle n|\hat{\mathbf{p}} \quad (3)$$

where Ze is the particle charge, and m is the mass of the particle (reduced mass in the alpha-decay case). We use the units $\hbar = c = 1$. Substituting the logarithm arising from the frequency integration by its average value $L = \ln(\beta m/\omega_{min})$, we can use the closure relations and obtain a simple expression

$$\hat{M}(E) = \frac{2Z^2\alpha}{3\pi m^2} L \hat{\mathbf{p}}(\hat{H} - E)\hat{\mathbf{p}} \quad (4)$$

$$= \frac{Z^2\alpha}{3\pi m^2} L \left\{ \nabla^2 \hat{U} + [(\hat{H} - E), \hat{\mathbf{p}}^2]_+ \right\}. \quad (5)$$

The mean value of the term with the anticommutator $[..., ...]_+$ in eq. (5) is equal to zero since $(\hat{H} - E)\Psi_0 = 0$ where Ψ_0 is the unperturbed wave function. A correction to the wave function due to this term can be calculated by using perturbation theory and the unperturbed Schrödinger equation,

$$\delta\Psi = \frac{2Z^2\alpha}{3\pi m} L [U - \langle 0|U|0\rangle] \Psi_0. \quad (6)$$

This correction is not essential since it does not influence the exponent in the tunneling amplitude.

Combining the remaining term in eq. (5) with the high-frequency contribution which contains $L = \ln(m/\beta m)$, see Ref. [5], the result can be presented as an effective local operator proportional to the Laplacian $\nabla^2 U(\mathbf{r})$,

$$M(\mathbf{r}, \mathbf{r}'; E) \simeq \nabla^2 U(\mathbf{r}) \delta(\mathbf{r} - \mathbf{r}') \frac{Z^2\alpha}{3\pi m^2} \ln \frac{m}{U_0} \equiv \delta U(\mathbf{r}) \delta(\mathbf{r} - \mathbf{r}'). \quad (7)$$

Here we used the barrier height U_0 as a lower cut-off ω_{min} of the integration over frequencies (below we give a semiclassical estimate which leads also to a more accurate evaluation of the logarithmic factor). For the tunneling of an extended object, the mass m in the argument of the logarithm should be replaced by the inverse size of the particle $1/r_0$ which comes from

the upper frequency cut-off given in this case by the charge formfactor. The obtained result is physically equivalent to the averaging over the position fluctuations due to the coupling to virtual photons. Thus, in the logarithmic approximation the mass operator is reduced to a local correction $\delta U(\mathbf{r})$ to the potential $U(\mathbf{r})$.

The Laplacian of the potential energy $\nabla^2 U(\mathbf{r})$ near the maximum of the barrier is negative (correspondingly, near the bottom of the potential well it is positive). Therefore, we obtained the negative correction $\delta U(\mathbf{r})$ to the potential barrier which leads to a conclusion that jiggling of the photon increases the tunneling amplitude of the particle. The numerical value of the correction to the potential is small, ~ 1 keV, for the alpha-decay. However, in some cases it may be noticeable due to the exponential dependence on the height of the barrier (recall the notorious cold fusion problem). Also, there exist theories like QCD where the radiation corrections are not small. In many-body systems one can use collective modes, as phonons, to transfer energy. This can influence electron tunneling through quantum dots or insulating surfaces. Energy exchange between a tunneling particle and nuclear environment is known to be important in subbarrier nuclear fission and fusion [4].

An analysis can be performed more in detail by using the semiclassical WKB approximation for the tunneling wave functions. The semiclassical radial Green function of unperturbed motion under the barrier can be written in terms of the classical momentum in the forbidden region, $p(r; E) = [2m(U(r) - E)]^{1/2}$, at a given energy E as

$$G(r, r'; E) = -m[p(r)p(r')]^{-1/2} \left\{ e^{-\int_r^r d\xi p(\xi)} \Theta(r - r') + e^{-\int_r^{r'} d\xi p(\xi)} \Theta(r' - r) \right\} \quad (8)$$

where $\Theta(x)$ is the step-function. The full three-dimensional Green function $G(E) = \sum_n |n\rangle (E - E_n)^{-1} \langle n|$ contains also angular harmonics which could be separated in a routine way accounting for the fact that in the long wavelength approximation for the s -wave solution the intermediate states are p -waves. Indeed, the operator of electric dipole radiation $\hat{\mathbf{p}}$ converts an initial s -wave Ψ in eq. (1) into an intermediate p -wave state $|n\rangle$. Therefore, it is sufficient to keep the p -wave part of the radial Green function and to use closure in the sum over angular harmonics.

The kernel of the integral term in the Schrödinger equation (1) contains

$$K(r, r'; E) = \int d\omega G(r, r'; E - \omega). \quad (9)$$

The integrand consists of terms falling exponentially as $|r - r'|$ increases. The potential $U(r)$ is assumed to be a smooth function. Therefore, we can put $p(r') \approx p(r)$. Now it is easy to perform the integration over ω in eq.(9) which leads to

$$K(r, r'; E) = -\frac{1}{|r - r'|} \left\{ e^{-p_{min}|r-r'|} - e^{-p_{max}|r-r'|} \right\} \quad (10)$$

where $p_{min} = [2m(U_p(r) - E)]^{1/2}$, $p_{max} = [2\beta]^{1/2}m$, and $U_p(r)$ is the effective p -wave radial potential which includes the centrifugal part. This expression has a very narrow maximum near $r = r'$ with the width $|r - r'| \sim 1/p_{max}$. This is a measure of non-locality of the self-energy operator $M(r, r'; E)$. In any nonrelativistic application the kernel can be treated as proportional to the delta-function. The proportionality coefficient can be found by the integration over r . Thus, we obtain the local behavior of the kernel,

$$K(r, r'; E) \approx -L(r)\delta(r - r'), \quad (11)$$

where now we determine the lower limit of the logarithm which has appeared in our previous derivation (7) as related to the local value of the potential,

$$L(r) = \ln \frac{m}{|U_p(r) - E|}. \quad (12)$$

The substitution into eq. (7) gives

$$\delta U(\mathbf{r}) = \frac{Z^2\alpha}{3\pi m^2} \ln \frac{m}{|U_p(r) - E|} \nabla^2 U(\mathbf{r}). \quad (13)$$

As usual, this semiclassical expression is not valid near the turning points where $U_p(r) = E$. However, a very weak logarithmic singularity does not produce any practical limitations on the applicability of eq. (13).

The conclusion of enhancement of the tunneling probability seems to contradict to the common sense: radiation should cause energy losses and reduce the tunneling amplitude

of the charged particle. However, such an argument may be valid only for the real photon emission. This emission is described by the antihermitian part of the self-energy operator which is originated from the delta-function corresponding to on-shell processes,

$$\Gamma(\mathbf{r}, \mathbf{r}'; E) = \frac{4Z^2\alpha}{3m^2} \int d\omega \sum_n \langle \mathbf{r} | \hat{\mathbf{p}} e^{i\mathbf{k}\mathbf{r}} | n \rangle \langle n | \hat{\mathbf{p}} e^{-i\mathbf{k}\mathbf{r}'} | \mathbf{r}' \rangle \omega \delta(E - E_n - \omega). \quad (14)$$

Because of the energy conservation the sum here includes only states $|n\rangle$ with energy E_n below E . Consider for example the tunneling from the ground s -state. A dipole transition transfers the particle from the s -state to a p -state. However, there are no quasidiscrete p -states $|n\rangle$ below the ground state in the potential well. Scattering p -waves can penetrate the potential barrier from the continuum with an exponentially small amplitude. This means that $\Gamma(\mathbf{r}, \mathbf{r}'; E)$ is again exponentially small if one or both arguments \mathbf{r} and \mathbf{r}' are under the barrier or inside the potential well. Therefore, $\Gamma(\mathbf{r}, \mathbf{r}'; E)$ does not considerably influence the tunneling amplitude. The reason for that can be easily understood. A real radiation would be impossible if there were no tunneling. Whence, the radiation width must vanish together with the tunneling width. On the contrary, the real part of Σ under the barrier would present even if the tunneling probability would vanish.

To avoid misunderstanding we need to stress that the contribution to the radiation intensity from the barrier area and the potential well, which was, in application to the nuclear alpha-decay, the subject of recent experimental [6] and theoretical [7–9] studies, still may be important. The radiation amplitude with $E_s - E_p = \omega$ contains the matrix element

$$\langle s | \hat{\mathbf{p}} | p \rangle = \frac{1}{\omega} \langle s | [\hat{H}, \hat{\mathbf{p}}] | p \rangle = \frac{i}{\omega} \langle s | \nabla \hat{U} | p \rangle. \quad (15)$$

When one moves inside the barrier from the outer turning point inwards, the resonance s -wave function exponentially increases while the non-resonance p -wave function exponentially decreases. As a result, the product $\psi_s(r)\psi_p(r)$ does not change considerably. This means that the contribution to the real radiation from the inner area may be comparable to that from the area outside the barrier. The gradient ∇U changes its sign near the maximum of the potential which implies a destructive interference between the radiation from the different

areas (since $|s\rangle$ is the non-oscillating ground state wave function, the product $\psi_s(r)\psi_p(r)$ does not change sign inside the barrier).

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